Solution

a. We are given that $\lambda = 550\text{ nm}$, $m = 2$, and $\theta_2 = 45.0°$. Solving the equation $D\sin\theta = m\lambda$ for D and substituting known values gives

$$D = \frac{m\lambda}{\sin\theta_2} = \frac{2(550\text{ nm})}{\sin 45.0°} = \frac{1100 \times 10^{-9}\text{ m}}{0.707} = 1.56 \times 10^{-6}\text{ m}.$$

b. Solving the equation $D\sin\theta = m\lambda$ for $\sin\theta_1$ and substituting the known values gives

$$\sin\theta_1 = \frac{m\lambda}{D} = \frac{1(550 \times 10^{-9}\text{ m})}{1.56 \times 10^{-6}\text{ m}}.$$

Thus the angle θ_1 is

$$\theta_1 = \sin^{-1} 0.354 = 20.7°.$$

Significance

We see that the slit is narrow (it is only a few times greater than the wavelength of light). This is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit significant wave effects such as this single-slit diffraction pattern. We also see that the central maximum extends $20.7°$ on either side of the original beam, for a width of about $41°$. The angle between the first and second minima is only about $24°$ $(45.0° - 20.7°)$. Thus, the second maximum is only about half as wide as the central maximum.

4.1 **Check Your Understanding** Suppose the slit width in **Example 4.1** is increased to 1.8×10^{-6} m. What are the new angular positions for the first, second, and third minima? Would a fourth minimum exist?

4.2 | Intensity in Single-Slit Diffraction

Learning Objectives

By the end of this section, you will be able to:

- Calculate the intensity relative to the central maximum of the single-slit diffraction peaks
- Calculate the intensity relative to the central maximum of an arbitrary point on the screen

To calculate the intensity of the diffraction pattern, we follow the phasor method used for calculations with ac circuits in **Alternating-Current Circuits (http://cnx.org/content/m58485/latest/)**. If we consider that there are N Huygens sources across the slit shown in **Figure 4.4**, with each source separated by a distance D/N from its adjacent neighbors, the path difference between waves from adjacent sources reaching the arbitrary point P on the screen is $(D/N)\sin\theta$. This distance is equivalent to a phase difference of $(2\pi D/\lambda N)\sin\theta$. The phasor diagram for the waves arriving at the point whose angular position is θ is shown in **Figure 4.7**. The amplitude of the phasor for each Huygens wavelet is ΔE_0, the amplitude of the resultant phasor is E, and the phase difference between the wavelets from the first and the last sources is

$$\phi = \left(\frac{2\pi}{\lambda}\right) D\sin\theta.$$

With $N \to \infty$, the phasor diagram approaches a circular arc of length $N\Delta E_0$ and radius r. Since the length of the arc is $N\Delta E_0$ for any ϕ, the radius r of the arc must decrease as ϕ increases (or equivalently, as the phasors form tighter spirals).

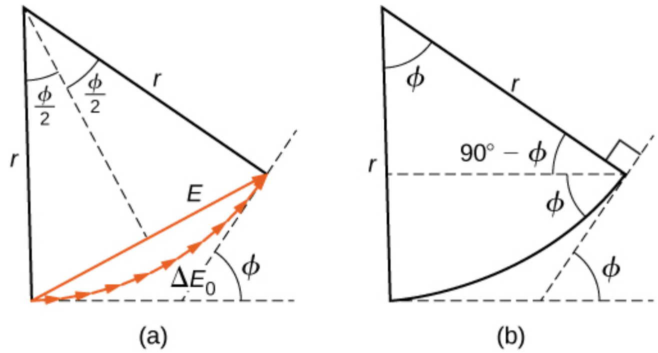

Figure 4.7 (a) Phasor diagram corresponding to the angular position θ in the single-slit diffraction pattern. The phase difference between the wavelets from the first and last sources is $\phi = (2\pi/\lambda)D \sin\theta$. (b) The geometry of the phasor diagram.

The phasor diagram for $\phi = 0$ (the center of the diffraction pattern) is shown in **Figure 4.8**(a) using $N = 30$. In this case, the phasors are laid end to end in a straight line of length $N\Delta E_0$, the radius *r* goes to infinity, and the resultant has its maximum value $E = N\Delta E_0$. The intensity of the light can be obtained using the relation $I = \frac{1}{2}c\varepsilon_0 E^2$ from **Electromagnetic Waves (http://cnx.org/content/m58495/latest/)** . The intensity of the maximum is then

$$I_0 = \frac{1}{2}c\varepsilon_0 (N\Delta E_0)^2 = \frac{1}{2\mu_0 c}(N\Delta E_0)^2,$$

where $\varepsilon_0 = 1/\mu_0 c^2$. The phasor diagrams for the first two zeros of the diffraction pattern are shown in parts (b) and (d) of the figure. In both cases, the phasors add to zero, after rotating through $\phi = 2\pi$ rad for $m = 1$ and 4π rad for $m = 2$.

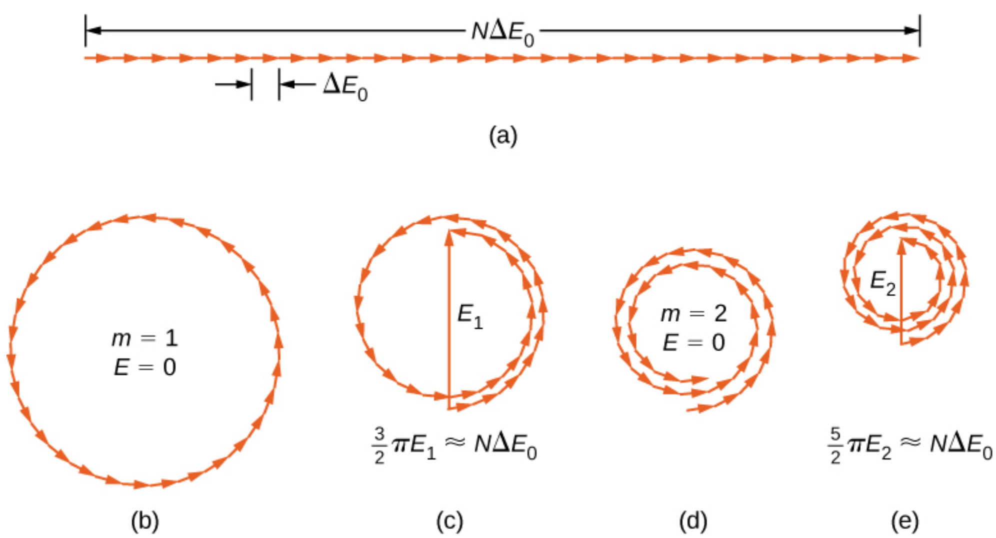

Figure 4.8 Phasor diagrams (with 30 phasors) for various points on the single-slit diffraction pattern. Multiple rotations around a given circle have been separated slightly so that the phasors can be seen. (a) Central maximum, (b) first minimum, (c) first maximum beyond central maximum, (d) second minimum, and (e) second maximum beyond central maximum.

The next two maxima beyond the central maxima are represented by the phasor diagrams of parts (c) and (e). In part (c), the phasors have rotated through $\phi = 3\pi$ rad and have formed a resultant phasor of magnitude E_1. The length of the arc formed by the phasors is $N\Delta E_0$. Since this corresponds to 1.5 rotations around a circle of diameter E_1, we have

$$\frac{3}{2}\pi E_1 \approx N\Delta E_0,$$

so

$$E_1 = \frac{2N\Delta E_0}{3\pi}$$

and

$$I_1 = \frac{1}{2\mu_0 c}E_1^2 = \frac{4(N\Delta E_0)^2}{(9\pi^2)(2\mu_0 c)} \approx 0.045 I_0,$$

where

$$I_0 = \frac{(N\Delta E_0)^2}{2\mu_0 c}.$$

In part (e), the phasors have rotated through $\phi = 5\pi$ rad, corresponding to 2.5 rotations around a circle of diameter E_2 and arc length $N\Delta E_0$. This results in $I_2 \approx 0.016 I_0$. The proof is left as an exercise for the student (**Exercise 4.119**).

These two maxima actually correspond to values of ϕ slightly less than 3π rad and 5π rad. Since the total length of the arc of the phasor diagram is always $N\Delta E_0$, the radius of the arc decreases as ϕ increases. As a result, E_1 and E_2 turn out to be slightly larger for arcs that have not quite curled through 3π rad and 5π rad, respectively. The exact values of ϕ for the maxima are investigated in **Exercise 4.120**. In solving that problem, you will find that they are less than, but very close to, $\phi = 3\pi, 5\pi, 7\pi, \ldots$ rad.

To calculate the intensity at an arbitrary point *P* on the screen, we return to the phasor diagram of **Figure 4.7**. Since the arc subtends an angle ϕ at the center of the circle,

$$N\Delta E_0 = r\phi$$

and

$$\sin\left(\frac{\phi}{2}\right) = \frac{E}{2r}.$$

where *E* is the amplitude of the resultant field. Solving the second equation for *E* and then substituting *r* from the first equation, we find

$$E = 2r\sin\frac{\phi}{2} = 2\frac{N\Delta E_o}{\phi}\sin\frac{\phi}{2}.$$

Now defining

$$\beta = \frac{\phi}{2} = \frac{\pi D\sin\theta}{\lambda} \quad (4.2)$$

we obtain

$$E = N\Delta E_0 \frac{\sin\beta}{\beta} \quad (4.3)$$

This equation relates the amplitude of the resultant field at any point in the diffraction pattern to the amplitude $N\Delta E_0$ at the central maximum. The intensity is proportional to the square of the amplitude, so

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \tag{4.4}$$

where $I_0 = (N \Delta E_0)^2 / 2\mu_0 c$ is the intensity at the center of the pattern.

For the central maximum, $\phi = 0$, β is also zero and we see from l'Hôpital's rule that $\lim_{\beta \to 0} (\sin \beta / \beta) = 1$, so that $\lim_{\phi \to 0} I = I_0$. For the next maximum, $\phi = 3\pi$ rad, we have $\beta = 3\pi/2$ rad and when substituted into **Equation 4.4**, it yields

$$I_1 = I_0 \left(\frac{\sin 3\pi/2}{3\pi/2} \right)^2 \approx 0.045 I_0,$$

in agreement with what we found earlier in this section using the diameters and circumferences of phasor diagrams. Substituting $\phi = 5\pi$ rad into **Equation 4.4** yields a similar result for I_2.

A plot of **Equation 4.4** is shown in **Figure 4.9** and directly below it is a photograph of an actual diffraction pattern. Notice that the central peak is much brighter than the others, and that the zeros of the pattern are located at those points where $\sin \beta = 0$, which occurs when $\beta = m\pi$ rad. This corresponds to

$$\frac{\pi D \sin \theta}{\lambda} = m\pi,$$

or

$$D \sin \theta = m\lambda,$$

which is **Equation 4.1**.

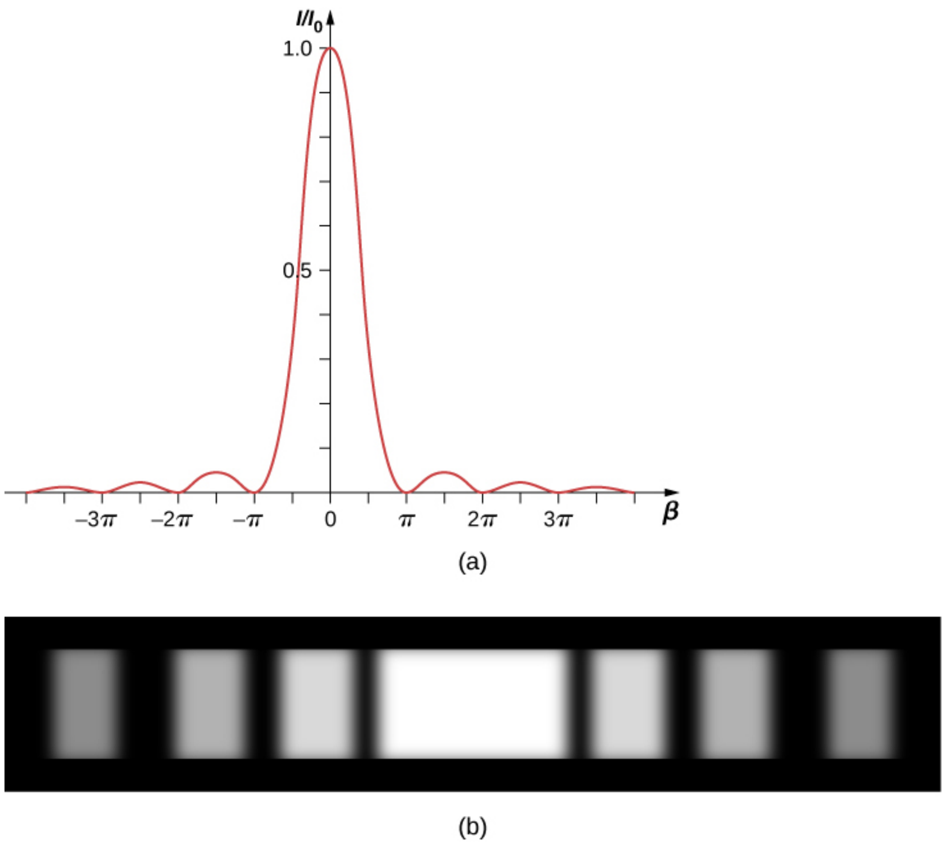

Figure 4.9 (a) The calculated intensity distribution of a single-slit diffraction pattern. (b) The actual diffraction pattern.

Example 4.2

Intensity in Single-Slit Diffraction

Light of wavelength 550 nm passes through a slit of width $2.00\ \mu\text{m}$ and produces a diffraction pattern similar to that shown in **Figure 4.9**. (a) Find the locations of the first two minima in terms of the angle from the central maximum and (b) determine the intensity relative to the central maximum at a point halfway between these two minima.

Strategy

The minima are given by **Equation 4.1**, $D\sin\theta = m\lambda$. The first two minima are for $m = 1$ and $m = 2$. **Equation 4.4** and **Equation 4.2** can be used to determine the intensity once the angle has been worked out.

Solution

a. Solving **Equation 4.1** for θ gives us $\theta_m = \sin^{-1}(m\lambda/D)$, so that

$$\theta_1 = \sin^{-1}\left(\frac{(+1)\left(550 \times 10^{-9}\ \text{m}\right)}{2.00 \times 10^{-6}\ \text{m}}\right) = +16.0°$$

and

$$\theta_2 = \sin^{-1}\left(\frac{(+2)\left(550 \times 10^{-9}\ \text{m}\right)}{2.00 \times 10^{-6}\ \text{m}}\right) = +33.4°.$$

b. The halfway point between θ_1 and θ_2 is

$$\theta = (\theta_1 + \theta_2)/2 = (16.0° + 33.4°)/2 = 24.7°.$$

Equation 4.2 gives

$$\beta = \frac{\pi D\sin\theta}{\lambda} = \frac{\pi\left(2.00 \times 10^{-6}\ \text{m}\right)\sin(24.7°)}{\left(550 \times 10^{-9}\ \text{m}\right)} = 1.52\pi \text{ or } 4.77 \text{ rad.}$$

From **Equation 4.4**, we can calculate

$$\frac{I}{I_o} = \left(\frac{\sin\beta}{\beta}\right)^2 = \left(\frac{\sin(4.77)}{4.77}\right)^2 = \left(\frac{-0.9985}{4.77}\right)^2 = 0.044.$$

Significance

This position, halfway between two minima, is very close to the location of the maximum, expected near $\beta = 3\pi/2$, or 1.5π.

4.2 Check Your Understanding For the experiment in **Example 4.2**, at what angle from the center is the third maximum and what is its intensity relative to the central maximum?

If the slit width D is varied, the intensity distribution changes, as illustrated in **Figure 4.10**. The central peak is distributed over the region from $\sin\theta = -\lambda/D$ to $\sin\theta = +\lambda/D$. For small θ, this corresponds to an angular width $\Delta\theta \approx 2\lambda/D$. Hence, an increase in the slit width results in a decrease in the **width of the central peak**. For a slit with $D \gg \lambda$, the central peak is very sharp, whereas if $D \approx \lambda$, it becomes quite broad.

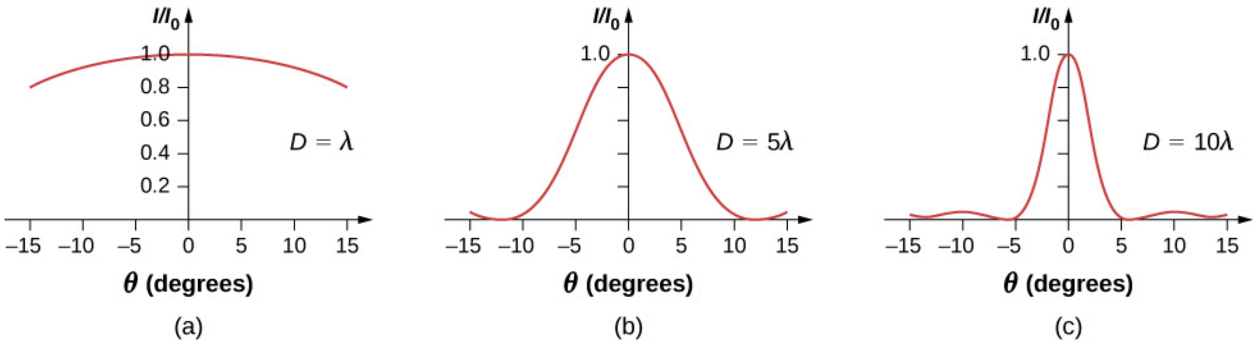

Figure 4.10 Single-slit diffraction patterns for various slit widths. As the slit width *D* increases from $D = \lambda$ to 5λ and then to 10λ, the width of the central peak decreases as the angles for the first minima decrease as predicted by **Equation 4.1**.

A diffraction experiment in optics can require a lot of preparation but **this simulation (https://openstaxcollege.org/l/21diffrexpoptsi)** by Andrew Duffy offers not only a quick set up but also the ability to change the slit width instantly. Run the simulation and select "Single slit." You can adjust the slit width and see the effect on the diffraction pattern on a screen and as a graph.

4.3 | Double-Slit Diffraction

Learning Objectives

By the end of this section, you will be able to:

- Describe the combined effect of interference and diffraction with two slits, each with finite width
- Determine the relative intensities of interference fringes within a diffraction pattern
- Identify missing orders, if any

When we studied interference in Young's double-slit experiment, we ignored the diffraction effect in each slit. We assumed that the slits were so narrow that on the screen you saw only the interference of light from just two point sources. If the slit is smaller than the wavelength, then **Figure 4.10**(a) shows that there is just a spreading of light and no peaks or troughs on the screen. Therefore, it was reasonable to leave out the diffraction effect in that chapter. However, if you make the slit wider, **Figure 4.10**(b) and (c) show that you cannot ignore diffraction. In this section, we study the complications to the double-slit experiment that arise when you also need to take into account the diffraction effect of each slit.

To calculate the diffraction pattern for two (or any number of) slits, we need to generalize the method we just used for a single slit. That is, across each slit, we place a uniform distribution of point sources that radiate Huygens wavelets, and then we sum the wavelets from all the slits. This gives the intensity at any point on the screen. Although the details of that calculation can be complicated, the final result is quite simple:

Two-Slit Diffraction Pattern

The diffraction pattern of two slits of width *D* that are separated by a distance *d* is the interference pattern of two point sources separated by *d* multiplied by the diffraction pattern of a slit of width *D*.

In other words, the *locations* of the interference fringes are given by the equation $d \sin\theta = m\lambda$, the same as when we considered the slits to be point sources, but the *intensities* of the fringes are now reduced by diffraction effects, according to **Equation 4.4**. [Note that in the chapter on interference, we wrote $d \sin\theta = m\lambda$ and used the integer *m* to refer to interference fringes. **Equation 4.1** also uses *m*, but this time to refer to diffraction minima. If both equations are used simultaneously, it is good practice to use a different variable (such as *n*) for one of these integers in order to keep them distinct.]

Interference and diffraction effects operate simultaneously and generally produce minima at different angles. This gives rise to a complicated pattern on the screen, in which some of the maxima of interference from the two slits are missing if the